PC-254
(512) M.A./M.Sc. Mathematics (SECOND SEMESTER)
Examination- JUNE-2020

Compulsory/Optional
Group -
Paper-II
REAL ANALYSIS(II)

Time:- Three Hours] **[Maximum Marks:80**

नोट : दोनो खण्डों से निर्देशानुसार उत्तर दीजिए। प्रश्नों के अंक उनके दाहिनी ओर अंकित है।
Note: Answer from Both the Section as Directed. The Figures in the right-hand margin indicated marks.

Section-A

1. Answer the following question: **1x10**

(a) For $A \leq R$, $m^*(A+x)$ = ---------- for all $x \in R$.
(b) The outer measure of A = [-1,1] is ----------.
(c) Write an example of measurable set.
(d) If $m^*(A) = 0$ then $m^*(A \cup B)$ = ----------.
(e) Every measurable function defined over the measurable set is continuous (True/False)
(f) If f is a measurable function on a measurable set E then $|f|$ is measurable function on E (True/False)
(g) Sum and Product of two simple functions are simple (True/False)
(h) Every simple function is step function (True/False).
(i) Write upper left Dini derivatives of function f.
(j) Every step function belongs to every L^p- space (True/False)

2. Answer the following question: **2x5**

(a) Prove that $m^*(A) = 0$ for every singleton set.
(b) Define step function with example.
(c) Define simple function with example.
(d) Define function of bounded Variation.
(e) let X = {0,16} and $f: x \to R$ be a function $f(x) = x^{-1/4}, x \in X$ prove that $f \in L'(X)$

Section-B

Answer any four question of the following: **15x4**

3. Prove that outer measure of an interval is its length.

4. Prove that a Borel measurable set is Lebesgue measurable.

5. (a) Prove that the characteristic function of a set E is measurable if and only if E is a measurable set.
(b) Show that a function is simple if and only if it is measurable and assumes only a finite number of values.

6. (a) Prove that the limit of a convergence sequence of measurable function is measurable.
(b) Show that any function defined on a set of measure zero is measurable.

7. (a) Show that-

$$\lim_{n\to\infty} \int_0^1 \frac{n\sqrt{x}}{1+n^2x^2}\,dx = 0$$

(b) Prove that a bounded measurable function on [a,b] is Lebesgue integrable over [a,b]

8. (a) State Jordan's Decomposition theorem.

(b) If f is absolutely continuous on [a,b] and f = 0, then f is a constant function.

9. If f and g are absolutely continuous function on [a,b] then choose the correct function.

(i) f - g (ii) f . g (iii) $\frac{f}{g}$ $(g \neq 0)$

10. (a) State and prove Minkowski's Inequality.

(b) If $f \in L^1$ and $g \in L^\infty$ them prove that $\int |fg| \leq \|f\|, \|g\|_\infty$